Applied Mathematics Worksheet-2

① $A = \begin{bmatrix} -1 & 1 & 3 \\ 1 & 0 & 5 \end{bmatrix}_{2\times3}$, $B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & 1 \end{bmatrix}_{3\times3}$, $C = \begin{bmatrix} 2 & 5 \\ -2 & 3 \\ 4 & 1 \end{bmatrix}_{3\times2}$ & $D = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}_{2\times2}$

i) $2A^t + 3C$

$\rightarrow 2A^t = 2\begin{bmatrix} -1 & 1 \\ 1 & 0 \\ 3 & 5 \end{bmatrix} = \begin{bmatrix} -2 & 2 \\ 2 & 0 \\ 6 & 10 \end{bmatrix} + 3\begin{bmatrix} 2 & 5 \\ -2 & 3 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 2 \\ 2 & 0 \\ 6 & 10 \end{bmatrix} + \begin{bmatrix} 6 & 15 \\ -6 & 9 \\ 12 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 17 \\ -4 & 9 \\ 18 & 13 \end{bmatrix}$

ii) $2A + BC - (DA)^t \rightarrow$ doesn't exist

iii) $5AC - D^2 + (I_2)^3$

$\rightarrow AC = \begin{bmatrix} -1 & 1 & 3 \\ 1 & 0 & 5 \end{bmatrix}\begin{bmatrix} 2 & 5 \\ -2 & 3 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} (-1)(2)+(1)(-2)+(3)(4) & (-1)(5)+(1)(3)+(3)(1) \\ (1)(2)+(0)(-2)+(5)(4) & (1)(5)+(0)(3)+(5)(1) \end{bmatrix}$

$= \begin{bmatrix} -2-2+12 & -5+3+3 \\ 2+0+20 & 5+0+5 \end{bmatrix} = \begin{bmatrix} 8 & 1 \\ 22 & 10 \end{bmatrix}$

$\rightarrow D^2 = D\times D = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 16 & 9 \\ 12 & 13 \end{bmatrix}$

$\rightarrow (I_2)^3 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\rightarrow 5AC - D^2 + (I_2)^3 = 5\begin{bmatrix} 8 & 1 \\ 22 & 10 \end{bmatrix} - \begin{bmatrix} 16 & 9 \\ 12 & 13 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 40 & 5 \\ 110 & 50 \end{bmatrix} - \begin{bmatrix} 16 & 9 \\ 12 & 13 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 25 & -4 \\ 98 & 38 \end{bmatrix}$

iv) $ABC - D$

$\rightarrow \begin{bmatrix} -1 & 1 & 3 \\ 1 & 0 & 5 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -1+2+9 & -2-1+3 & -3+4+3 \\ 1+0+15 & 2+0+5 & 3+0+5 \end{bmatrix}$

$AB = \begin{bmatrix} 10 & 0 & 4 \\ 16 & 7 & 8 \end{bmatrix}_{2\times3}$

$\rightarrow ABC = \begin{bmatrix} 10 & 0 & 4 \\ 16 & 7 & 8 \end{bmatrix}\begin{bmatrix} 2 & 5 \\ -2 & 3 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 20+0+16 & 50+0+4 \\ 32-14+32 & 80+21+8 \end{bmatrix}$

$ABC = \begin{bmatrix} 36 & 54 \\ 50 & 109 \end{bmatrix}_{2\times2}$ $\Rightarrow ABC - D = \begin{bmatrix} 36 & 54 \\ 50 & 109 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$

$= \begin{bmatrix} 34 & 51 \\ 46 & 108 \end{bmatrix}$

② i) $\begin{bmatrix} 4 & 2x \\ -4x & -3 \end{bmatrix} + \begin{bmatrix} -5 & -3y \\ 5y & 3 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 4-5 & 2x-3y \\ -4x+5y & -3+3 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} -1 & 2x-3y \\ -4x+5y & 0 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix}$

$\Rightarrow \begin{cases} -4x+5y=1 \\ 2)(2x-3y=1) \end{cases}$ → $2x-3y=1$ ⇒ $\underline{\underline{x=-4 \;\&\; y=-3}}$

$2x-3(-3)=1$

$+\begin{cases} -\cancel{4x}+5y=1 \\ \cancel{4x}-6y=2 \end{cases}$ $2x+9=1$

$2x=-8$

$-y=3$ $\underline{\underline{x=-4}}$

$\underline{\underline{y=-3}}$

ii) $\begin{bmatrix} 3 & -1 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 1 \end{bmatrix}$ → $\begin{cases} (3x-y=10)5 \\ 2x+5y=1 \end{cases}$ ⇒ $+\begin{cases} 15x-5y=50 \\ 2x+5y=1 \end{cases}$

$17x=51$

$\underline{\underline{x=3}}$

→ $2x+5y=1$ ⇒ $\underline{\underline{x=3}} \;\&\; \underline{\underline{y=-1}}$

$6+5y=1$

$5y=-5$

$\underline{\underline{y=-1}}$

③ $A+B = \begin{bmatrix} 1 & 3 & 5 \\ 1 & -4 & 2 \\ 1 & 8 & 6 \end{bmatrix}$ & $A-B = \begin{bmatrix} 1 & 1 & 1 \\ 3 & -4 & 8 \\ 5 & 2 & 6 \end{bmatrix}$

→ $(A+B)+(A-B) = 2A = \begin{bmatrix} 2 & 4 & 6 \\ 4 & -8 & 10 \\ 6 & 10 & 12 \end{bmatrix}$ → $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$

$\cancel{\Rightarrow \text{let } B = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \Rightarrow \begin{bmatrix} 2+a & 4+b & 6+c \\ 4+d & -8+e & 10+f \\ 6+g & 10+h & 12+i \end{bmatrix} = \begin{bmatrix} 1 & 3 & 5 \\ 1 & -4 & 2 \\ 1 & 8 & 6 \end{bmatrix}}$

$\cancel{\Rightarrow \begin{bmatrix} 2-a & 4-b & 6-c \\ 4-d & -8-e & 10-f \\ 6-g & 10-h & 12-i \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 3 & -4 & 8 \\ 5 & 2 & 6 \end{bmatrix}}$

$\cancel{\Rightarrow 2+a=1}$

$\cancel{2-a=1}$

$\Rightarrow$ Let $B = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ $\Rightarrow A+B = \begin{bmatrix} 1+a & 2+b & 3+c \\ 2+d & -4+e & 5+f \\ 3+g & 5+h & 6+i \end{bmatrix} = \begin{bmatrix} 1 & 3 & 5 \\ 1 & -4 & 2 \\ 1 & 8 & 6 \end{bmatrix}$

$\Rightarrow A-B = \begin{bmatrix} 1-a & 2-b & 3-c \\ 2-d & -4-e & 5-f \\ 3-g & 5-h & 6-i \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 3 & -4 & 8 \\ 5 & 2 & 6 \end{bmatrix}$

$\Rightarrow 1+a = 1$, $\underline{\underline{a=0}}$
$1-a = 1$

$\rightarrow 2-b = 1$, $\underline{\underline{b=1}}$
$2+b = 3$

$\rightarrow 3+c = 5$, $\underline{\underline{c=2}}$
$3-c = 1$

$\rightarrow 2+d = 1$, $\underline{\underline{d=-1}}$
$2-d = 3$

$\rightarrow -4+e = -4$, $\underline{\underline{e=0}}$
$-4-e = -4$

$\rightarrow 5+f = 2$, $\underline{\underline{f=-3}}$
$5-f = 8$

$\rightarrow 3+g = 1$, $\underline{\underline{g=-2}}$
$3-g = 5$

$\rightarrow 5+h = 8$, $\underline{\underline{h=3}}$
$5-h = 2$

$\rightarrow 6+i = 6$, $\underline{\underline{i=0}}$
$6-i = 6$

$B = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{bmatrix}$ & $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$

(4) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}_{3\times3} A_{i\times j} \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}_{2\times2} = \begin{bmatrix} 0 & 1 \\ 5 & 8 \\ 0 & 0 \end{bmatrix}_{3\times2}$

$\cancel{A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 0 & 1 \\ 5 & 8 \\ 0 & 0 \end{bmatrix}}$

$\cancel{\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}^{-1} \Rightarrow \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{vmatrix} = -1 \quad C_{ij} = [\ ]}$

$\Rightarrow (3\times3)(i\times j)(2\times2) = 3\times2$ $\Rightarrow$ The order of A is 3×2
$\underline{\underline{i=3}}$ $\Rightarrow (3\times j)(2\times2) = 3\times2$ $\quad \underline{\underline{A_{3\times2}}}$
$\underline{\underline{j=2}}$

$\rightarrow$ Let $A = \begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 5 & 8 \\ 0 & 0 \end{bmatrix}$

$\rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}_{3\times3} \begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}_{3\times2} = \begin{bmatrix} a & b \\ c & d \\ e & -f \end{bmatrix} \Rightarrow \begin{bmatrix} a & b \\ c & d \\ e & -f \end{bmatrix}_{3\times2} \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}_{2\times2} = \begin{bmatrix} 0 & 1 \\ 5 & 8 \\ 0 & 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} a+b & a+2b \\ c+d & c+2d \\ e-f & e-2f \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 5 & 8 \\ 0 & 0 \end{bmatrix} \Rightarrow \begin{cases} a+b=0 \\ a+2b=1 \end{cases}$

$-b=-1 \quad a=-b$

$b=1 \quad a=-1$

$\Rightarrow \begin{cases} c+d=5 \\ c+2d=8 \end{cases}$

$-d=-3 \quad c+3=5$

$d=3 \quad c=2$

$\Rightarrow \begin{cases} e-f=0 \\ e-2f=0 \end{cases}$

$f=0 \quad e=f$

$e=0$

$\Rightarrow A = \begin{bmatrix} -1 & 1 \\ 2 & 3 \\ 0 & 0 \end{bmatrix}_{3\times2}$

(5) i) $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} \rightarrow$ symmetric

ii) $\begin{bmatrix} -1 & 0 & 1 \\ 0 & 2 & 2 \\ 2 & 1 & -1 \end{bmatrix} \rightarrow$ neither

iii) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & -4 & 5 \\ 3 & 5 & 6 \end{bmatrix} \rightarrow$ symmetric

iv) $\begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ 2 & -3 & 0 \end{bmatrix} \rightarrow$ skew-symmetric

(6) $A = \begin{bmatrix} 3 & 0 & 1 \\ 4 & 2 & 5 \\ 7 & 3 & 1 \end{bmatrix}, \quad A^t = \begin{bmatrix} 3 & 4 & 7 \\ 0 & 2 & 3 \\ 1 & 5 & 1 \end{bmatrix}$

$\rightarrow \cancel{A+A}\ A = \frac{(A+A^t)+(A-A^t)}{2} \rightarrow A+A^t = \begin{bmatrix} 6 & 4 & 8 \\ 4 & 4 & 8 \\ 8 & 8 & 2 \end{bmatrix}$

$\rightarrow A-A^t = \begin{bmatrix} 0 & -4 & -6 \\ 4 & 0 & 2 \\ 6 & -2 & 0 \end{bmatrix}$

$$\Rightarrow A = \frac{(A+A^t)+(A-A^t)}{2} = \frac{\begin{bmatrix} 6 & 4 & 8 \\ 4 & 4 & 8 \\ 8 & 8 & 2 \end{bmatrix} + \begin{bmatrix} 0 & -4 & -6 \\ 4 & 0 & 2 \\ 6 & -2 & 0 \end{bmatrix}}{2}$$

symmetric — Skew-symmetric

(7) i) $\begin{vmatrix} 0 & 1 & 4 \\ 2 & 3 & 0 \\ -1 & 0 & 5 \end{vmatrix} = -1(10)+4(3)$

$= -10+12 = 2$

ii) $\begin{vmatrix} x+1 & 2 & 3 \\ 0 & x+2 & 1 \\ -1 & 0 & x \end{vmatrix} \Rightarrow$ Using row 3

~~$= 1(2-(3x+6)) + x((x+1)(x+2))$~~

~~$= 1(2-3x-6) + x(x^2+2x+x+2)$~~

~~$= -4-3x + x(x^2+3x+2)$~~

~~$= -4-3x+x^3+3x^2+2x = x^3+3x^2-x-4$~~

$\Rightarrow$ ~~$1((x+1)(x+2)-0)+x($~~

$= 1(2-(3x+6)) + x((x-1)(x+2))$

$= 1(2-3x-6) + x(x^2+2x-x-2)$

$= -4-3x+x^3+x^2-2x$

$= x^3+x^2-5x-4$

iii) $\begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix} = \cos^2\theta + \sin^2\theta = 1$

iv) $\begin{vmatrix} 2 & 3 & -2 & 3 \\ 1 & 0 & 2 & 1 \\ -1 & 1 & 4 & -2 \\ 3 & 0 & 0 & 4 \end{vmatrix} \Rightarrow 2\begin{vmatrix} 0 & 2 & 1 \\ 1 & 4 & -2 \\ 0 & 0 & 4 \end{vmatrix} - 3\begin{vmatrix} 1 & 2 & 1 \\ -1 & 4 & -2 \\ 3 & 0 & 4 \end{vmatrix}$

$- 2\begin{vmatrix} 1 & 0 & 1 \\ -1 & 1 & -2 \\ 3 & 0 & 4 \end{vmatrix} - 3\begin{vmatrix} 1 & 0 & 2 \\ -1 & 1 & 4 \\ 3 & 0 & 0 \end{vmatrix}$

$\Rightarrow -2\left[-2(4)+1(0)\right]-3\left[1(16)-2(2)+1(-12)\right]$

$-2\left[1(4)+1(-3)\right]-3\left[1(0)+2(-3)\right]$

$\Rightarrow -2(-8+0)-3(16-4-12)-2(4-3)-3(0-6)$

$= -2(-8)-3(0)-2(1)-3(-6) = 16-2+18 = \underline{\underline{32}}$

(8) $\begin{bmatrix} 1 & x+1 & 0 \\ 1 & 0 & -1 \\ 1 & 4 & x \end{bmatrix} \Rightarrow \begin{vmatrix} 1 & x+1 & 0 \\ 1 & 0 & -1 \\ 1 & 4 & x \end{vmatrix} = 0$

$\Rightarrow 1(4)-(x+1)(x+1)=0$

$\Rightarrow 4-(x+1)^2=0$

$\Rightarrow (x+1)^2=4$

$\Rightarrow x+1=\pm 2$

$x=\pm 2-1$

$\underline{\underline{x=1}}$ or $\underline{\underline{x=-3}}$

(9) $[A]_{4\times4}$ $[B]_{4\times4}$, $\det(A)=-4$, $\det(B)=2$

$\det(2AB^tA^{-1})$

$\Rightarrow |2AB^tA^{-1}| = 2^4|AB^tA^{-1}| = 16|A||B^t||A^{-1}|$

$= 16|A||B|\frac{1}{|A|}$

$= (16)(-4)(2)\frac{1}{(-4)} = \underline{\underline{32}}$

(10) If $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 10$ then $\begin{vmatrix} a & b & c \\ 2g & 2h & 2i \\ 3d-a & 3e-b & 3f-c \end{vmatrix} = ?$

$\Rightarrow \begin{vmatrix} a & b & c \\ 2g & 2h & 2i \\ 3d-a & 3e-b & 3f-c \end{vmatrix} = -(2)(3)(10) = \underline{\underline{-60}}$

(12) i) $\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix} \rightarrow \begin{vmatrix} 2 & 3 \\ 3 & 4 \end{vmatrix} = -1$

$adj = C_{2\times2}^t = \begin{bmatrix} 4 & -3 \\ -3 & 2 \end{bmatrix}$

$\Rightarrow$ Inverse $= \dfrac{\begin{bmatrix} 4 & -3 \\ -3 & 2 \end{bmatrix}}{-1} = \underline{\underline{\begin{bmatrix} -4 & 3 \\ 3 & -2 \end{bmatrix}}}$

ii) let $B = \begin{bmatrix} 3 & 6 \\ 4 & 8 \end{bmatrix} \rightarrow |B| = 0 \rightarrow$ not invertible

iii)

$$\text{let } C = \begin{bmatrix} 1 & 2 & -3 \\ 1 & 3 & 1 \\ 1 & 1 & -7 \end{bmatrix} \rightarrow |C| = 1(-21-1) - 2(-7-1) - 3(1-3)$$

$$= 1(-22) - 2(-8) - 3(-2)$$

$$= -22 + 16 + 6 = 0$$

$\rightarrow$ not invertible

iv) let $D = \begin{bmatrix} 3 & -4 & 1 \\ 2 & -3 & 1 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow |D| = 1(-9+8) = -1$

$$\rightarrow M_D = \begin{bmatrix} -3 & 2 & 0 \\ -4 & 3 & 0 \\ -1 & 1 & -1 \end{bmatrix} \rightarrow C_D = \begin{bmatrix} -3 & -2 & 0 \\ 4 & 3 & 0 \\ -1 & -1 & -1 \end{bmatrix}$$

$$\rightarrow \cancel{D^{-1}} = adj(D) = \begin{bmatrix} -3 & 4 & -1 \\ -2 & 3 & -1 \\ 0 & 0 & -1 \end{bmatrix} \rightarrow D^{-1} = \frac{adj(D)}{|D|} = \frac{\begin{bmatrix} -3 & 4 & -1 \\ -2 & 3 & -1 \\ 0 & 0 & -1 \end{bmatrix}}{-1}$$

$$\rightarrow D^{-1} = \begin{bmatrix} 3 & -4 & 1 \\ 2 & -3 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

13) i)

$$\begin{bmatrix} 2 & 3 & -2 & 9 \\ 5 & 7 & 1 & 8 \\ 3 & 4 & 2 & -1 \end{bmatrix} \rightarrow R_2 \rightarrow R_2 - R_1 \begin{bmatrix} 2 & 3 & -2 & 9 \\ 3 & 4 & 3 & -1 \\ 3 & 4 & 2 & -1 \end{bmatrix}$$

$$R_3 \rightarrow R_3 - R_2 \begin{bmatrix} 2 & 3 & -2 & 9 \\ 3 & 4 & 3 & -1 \\ 0 & 0 & -1 & 0 \end{bmatrix} \quad R_2 \rightarrow R_2 + 3R_3 \begin{bmatrix} 2 & 3 & -2 & 9 \\ 3 & 4 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

$$R_2 \rightarrow R_2 - R_1 \begin{bmatrix} 2 & 3 & -2 & 9 \\ 1 & 1 & 2 & -10 \\ 0 & 0 & -1 & 0 \end{bmatrix} \quad R_1 \rightarrow R_1 - R_2 \begin{bmatrix} 1 & 2 & -4 & 19 \\ 1 & 1 & 2 & -10 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

$$R_2 \rightarrow R_2 - R_1 \begin{bmatrix} 1 & 2 & -4 & 19 \\ 0 & -1 & 6 & -29 \\ 0 & 0 & -1 & 0 \end{bmatrix} \quad R_1 = R_1 + 2R_2 \begin{bmatrix} 1 & 0 & 8 & -39 \\ 0 & -1 & 6 & -29 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

$R_2 \to -R_2$ $\begin{bmatrix} 1 & 0 & 8 & -39 \\ 0 & 1 & -6 & 29 \\ 0 & 0 & -1 & 0 \end{bmatrix}$

$R_1 \to R_1 + 8R_3$ $\begin{bmatrix} 1 & 0 & 0 & -39 \\ 0 & 1 & -6 & 29 \\ 0 & 0 & -1 & 0 \end{bmatrix}$

$R_3 \to -R_3$ $\begin{bmatrix} 1 & 0 & 0 & -39 \\ 0 & 1 & -6 & 29 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ $R_2 \to R_2 + 6R_3$ $\begin{bmatrix} 1 & 0 & 0 & -39 \\ 0 & 1 & 0 & 29 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

$\Rightarrow$ rank = 3

ii) $\begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 6 \\ 5 & 8 & 6 \\ -2 & -8 & 0 \\ -1 & 0 & -4 \end{bmatrix}$ $R_2 \to R_2 - 3R_1$ $\begin{bmatrix} 1 & 2 & 1 \\ 0 & -2 & 3 \\ 5 & 8 & 6 \\ -2 & -8 & 0 \\ -1 & 0 & -4 \end{bmatrix}$

$R_2 \leftrightarrow R_4$ $\begin{bmatrix} 1 & 2 & 1 \\ -2 & -8 & 0 \\ 5 & 8 & 6 \\ 0 & -2 & 3 \\ -1 & 0 & -4 \end{bmatrix}$ $\Rightarrow R_2 \to R_2 + 2R_1$ $\begin{bmatrix} 1 & 2 & 1 \\ 0 & -4 & 2 \\ 5 & 8 & 6 \\ 0 & -2 & 3 \\ -1 & 0 & -4 \end{bmatrix}$

$R_2 \to R_2 + R_3$ $\begin{bmatrix} 1 & 2 & 1 \\ 5 & 4 & 8 \\ 5 & 8 & 6 \\ 0 & -2 & 3 \\ -1 & 0 & -4 \end{bmatrix}$ $\Rightarrow R_2 \to R_2 + 5R_5$ $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 4 & -12 \\ 5 & 8 & 6 \\ 0 & -2 & 3 \\ -1 & 0 & -4 \end{bmatrix}$

$R_2 \to \frac{1}{4} R_2$ $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -3 \\ 5 & 8 & 6 \\ 0 & -2 & 3 \\ -1 & 0 & -4 \end{bmatrix}$ $R_5 \to R_1 + R_5$ $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -3 \\ 5 & 8 & 6 \\ 0 & -2 & 3 \\ 0 & 2 & -3 \end{bmatrix}$

$R_4 \to R_4 + R_5$ $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -3 \\ 5 & 8 & 6 \\ 0 & 0 & 0 \\ 0 & 2 & -3 \end{bmatrix}$ $R_3 \leftrightarrow R_5$ $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -3 \\ 0 & 2 & -3 \\ 0 & 0 & 0 \\ 5 & 8 & 6 \end{bmatrix}$

$R_3 \to R_3 - R_1$ $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -3 \\ -1 & 0 & -4 \\ 0 & 0 & 0 \\ 5 & 8 & 6 \end{bmatrix}$ $R_1 \to R_1 - 2R_2$ $\begin{bmatrix} 1 & 0 & 7 \\ 0 & 1 & -3 \\ -1 & 0 & -4 \\ 0 & 0 & 0 \\ 5 & 8 & 6 \end{bmatrix}$

$$R_3 \to R_3 + R_1 \begin{bmatrix} 1 & 0 & 7 \\ 0 & 1 & -3 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \\ 5 & 8 & 6 \end{bmatrix} R_3 \to \frac{1}{3} R_3 \begin{bmatrix} 1 & 0 & 7 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 5 & 8 & 6 \end{bmatrix}$$

$$\xrightarrow[R_4 \leftrightarrow R_5]{R_1 \to R_1 - 7R_3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \\ 5 & 8 & 6 \\ 0 & 0 & 0 \end{bmatrix} R_2 \to R_2 + 3R_3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 5 & 8 & 6 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\xrightarrow[R_4 \to \frac{1}{2} R_4]{R_4 \to R_4 - R_1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 2 & 4 & 3 \\ 0 & 0 & 0 \end{bmatrix} R_4 \to R_4 - R_3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 2 & 4 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$R_4 \to \frac{1}{2} R_4 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix} R_4 \to R_4 - R_1 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\xrightarrow[R_4 \to R_4 - R_3]{R_4 \to R_4 - 2R_2} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

6 5 10
3 3 6
0 -1 -2

iii) $$\begin{bmatrix} 3 & 3 & 6 \\ 6 & 5 & 10 \\ 7 & 8 & 9 \end{bmatrix} \xrightarrow[R_1 \to R_1 - 2R_2]{R_2 \leftrightarrow R_1} \begin{bmatrix} 0 & -1 & -2 \\ 3 & 3 & 6 \\ 7 & 8 & 9 \end{bmatrix} R_3 \to R_3 - 2R_2 \begin{bmatrix} 0 & -1 & -2 \\ 3 & 3 & 6 \\ 1 & 2 & -3 \end{bmatrix}$$

$$\xrightarrow[R_2 \to \frac{1}{3} R_2]{R_3 \to R_3 + 2R_1} \begin{bmatrix} 0 & -1 & -2 \\ 1 & 1 & 2 \\ 1 & 0 & -7 \end{bmatrix} \xrightarrow[R_2 \to R_2 - R_3]{R_1 \to R_1 + R_2} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 9 \\ 1 & 0 & -7 \end{bmatrix}$$

$$R_3 \to R_1 - R_3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 9 \\ 0 & 0 & 7 \end{bmatrix} \xrightarrow[R_2 \to R_2 - 9R_3]{R_3 \to \frac{1}{7} R_3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(14) i) $x_1 - x_2 = 3$
$2x_1 + kx_2 = 6$

$$\left[\begin{array}{cc|c} 1 & -1 & 3 \\ 2 & k & 6 \end{array}\right] \Rightarrow R_2 \to R_2 - 2R_1 \left[\begin{array}{cc|c} 1 & -1 & 3 \\ 0 & k+2 & 0 \end{array}\right]$$

a) one sol$\underline{n}$ → impossible
b) no sol$\underline{n}$, when $k+2 \neq 0$ → $k \neq -2$ → $R / \{-2\}$
c) infinite → when $k+2 = 0$ → $k = 2$

ii) $x_1 + 2x_2 = 7$
$x_1 + (k^2+1)x_2 = 3$

$$\left[\begin{array}{cc|c} 1 & 2 & 7 \\ 1 & k^2+1 & 3 \end{array}\right] R_2 \to R_2 - R_1 \left[\begin{array}{cc|c} 1 & 2 & 7 \\ 0 & k^2-1 & -4 \end{array}\right]$$

a) one ~~sto~~ sol$\underline{n}$ → when $k^2-1 \neq 0$, $(k+1)(k-1) \neq 0$
$k \neq -1, 1$ → $R / \{-1, 1\}$

b) no sol$\underline{n}$, when $k = -1, 1$
c) infinite ~~when~~ → impossible

$\frac{\frac{3}{2}k - 3}{6}$ $\quad \frac{3k-6}{12}$ $\quad \frac{12-k}{3} + \frac{3k-42}{12}$

$\frac{3k-6}{12} - 3$ $\quad \frac{3k-42}{12}$ $\quad \frac{48-4k+3k-42}{12}$

$\frac{6-k}{12}$

$-\frac{2}{3} - \frac{13}{3}$

$-\frac{15}{3}$

iii) $x_1 + 2x_2 + 3x_3 = 4$
$4x_1 + 5x_2 + kx_3 = 3$
$7x_1 + 8x_2 + \frac{3}{2}kx_3 = 0$

$$\to \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 4 & 5 & k & 3 \\ 7 & 8 & \frac{3}{2}k & 0 \end{array}\right] R_2 \to R_2 - 4R_1 \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & -3 & k-12 & -13 \\ 7 & 8 & \frac{3}{2}k & 0 \end{array}\right]$$

$R_3 \to \cancel{\frac{1}{6}} R_3 - R_1$
$R_3 \to \frac{1}{6} R_3$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & -3 & k-12 & -13 \\ 1 & 1 & \frac{3k-6}{12} & -\frac{2}{3} \end{array}\right] R_2 \to -\frac{1}{3}R_2 \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & \frac{12-k}{3} & \frac{13}{3} \\ 1 & 1 & \frac{3k-6}{12} & -\frac{2}{3} \end{array}\right]$$

$$R_3 \to R_3 - R_1 \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & \frac{12-k}{3} & \frac{13}{3} \\ 0 & -1 & \frac{3k-42}{12} & -\frac{2}{3} \end{array}\right] R_3 \to R_2 + R_3 \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & \frac{12-k}{3} & \frac{13}{3} \\ 0 & 0 & \frac{6-k}{12} & -5 \end{array}\right]$$

a) one soln, when $\frac{6-k}{12} \neq 0$, $6-k \neq 0 \Rightarrow k \neq 6$

$\mathbb{R} / \{6\}$

b) no soln when $k = 6$

c) infinite soln $\to$ impossible

(15) i) $2x + y - z = 1$
$x - 2y + z = 5$
$3x - y - 2z = 0$

$\to$ a) Gauss elimination

$$\left[\begin{array}{ccc|c} 2 & 1 & -1 & 1 \\ 1 & -2 & 1 & 5 \\ 3 & -1 & -2 & 0 \end{array}\right]$$

$$\xrightarrow[R_3 \leftrightarrow R_2]{R_3 \leftrightarrow R_1} \left[\begin{array}{ccc|c} 3 & -1 & -2 & 0 \\ 2 & 1 & -1 & 1 \\ 1 & -2 & 1 & 5 \end{array}\right] \quad R_1 \to R_1 - R_2 \left[\begin{array}{ccc|c} 1 & -2 & -1 & -1 \\ 2 & 1 & -1 & 1 \\ 1 & -2 & 1 & 5 \end{array}\right]$$

$$R_3 \to R_3 - R_1 \left[\begin{array}{ccc|c} 1 & -2 & -1 & -1 \\ 2 & 1 & -1 & 1 \\ 0 & 0 & 2 & 6 \end{array}\right] \xrightarrow[R_3 \to \frac{1}{2} R_3]{R_2 \to R_2 - 2R_1} \left[\begin{array}{ccc|c} 1 & -2 & -1 & -1 \\ 0 & 5 & 1 & 3 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$$R_2 \to \frac{1}{5} R_2 \left[\begin{array}{ccc|c} 1 & -2 & -1 & -1 \\ 0 & 1 & \frac{1}{5} & \frac{3}{5} \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$\Rightarrow z = 3$

$y + \frac{1}{5}(3) = \frac{3}{5}$

$y + \frac{3}{5} = \frac{3}{5}$

$y = 0$

$x - \cancel{2(0)} - 1(3) = -1$

$x = -1 + 3$

$x = 2$

$\frac{1}{5} + \frac{9}{5}$

b) Gauss-Jordan elimination

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & -1 \\ 0 & 1 & \frac{1}{5} & \frac{3}{5} \\ 0 & 0 & 1 & 3 \end{array}\right] \quad R_1 \to R_1 + 2R_2 \left[\begin{array}{ccc|c} 1 & 0 & -\frac{3}{5} & \frac{1}{5} \\ 0 & 1 & \frac{1}{5} & \frac{3}{5} \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$\frac{3}{5} - \frac{3}{5}$

$$R_1 \to R_1 + \frac{3}{5} R_3 \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & \frac{1}{5} & \frac{3}{5} \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$$R_2 \to R_2 - \frac{1}{5} R_3 \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 \end{array}\right] \Rightarrow \begin{array}{l} x = 2 \\ y = 0 \\ z = 3 \end{array}$$

ii) $2x_1 - 4x_2 + x_3 = -4$
$4x_1 - 8x_2 + 7x_3 = 2$
$-2x_1 + 4x_2 - 3x_3 = 5$ $\rightarrow \left[\begin{array}{ccc|c} 2 & -4 & 1 & -4 \\ 4 & -8 & 7 & 2 \\ -2 & 4 & -3 & 5 \end{array}\right]$

$2(24-28)$
$+4(-12+14)$
$+1(\cancel{0})$
$2(-4)+4(2)$
$-8+8$

→ a) Gauss elimination

→ $R_2 \rightarrow R_2 - 2R_1$ $\left[\begin{array}{ccc|c} 2 & -4 & 1 & -4 \\ 0 & 0 & 5 & 10 \\ -2 & 4 & -3 & 5 \end{array}\right]$

$R_2 \rightarrow R_3$ $\left[\begin{array}{ccc|c} 2 & -4 & 1 & -4 \\ -2 & 4 & -3 & 5 \\ 0 & 0 & 5 & 10 \end{array}\right] \xrightarrow[R_1 \rightarrow \frac{1}{2}R_1]{R_3 \rightarrow \frac{1}{5}R_3} \left[\begin{array}{ccc|c} 1 & -2 & \frac{1}{2} & -2 \\ -2 & 4 & -3 & 5 \\ 0 & 0 & 1 & 2 \end{array}\right]$

$R_2 \rightarrow \frac{1}{2}R_2$ $\left[\begin{array}{ccc|c} 1 & -2 & \frac{1}{2} & -2 \\ -1 & 2 & -\frac{3}{2} & \frac{5}{2} \\ 0 & 0 & 1 & 2 \end{array}\right]$ $R_2 \rightarrow R_2 + R_1$ $\left[\begin{array}{ccc|c} 1 & -2 & \frac{1}{2} & -2 \\ 0 & 0 & -1 & \frac{1}{2} \\ 0 & 0 & 1 & 2 \end{array}\right]$

$R_3 \rightarrow R_3 + R_2$ $\left[\begin{array}{ccc|c} 1 & -2 & \frac{1}{2} & -2 \\ 0 & 0 & -1 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{5}{2} \end{array}\right]$ $\frac{5}{2} = 2 + \frac{1}{2}$ $\Rightarrow$ No solution

16) i) $x + y = 7$
$2x + 3y = 18$ $\rightarrow \left[\begin{array}{cc|c} 1 & 1 & 7 \\ 2 & 3 & 18 \end{array}\right]$

a) Matrix inverse method

$\begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 18 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}^{-1} \begin{bmatrix} 7 \\ 18 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \dfrac{\begin{bmatrix} 3 & -1 \\ -2 & 1 \end{bmatrix}}{1} \begin{bmatrix} 7 \\ 18 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 7 \\ 18 \end{bmatrix}$ $\rightarrow x = 3$, $y = 4$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 21-18 \\ -14+18 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

b) Cramer's Rule

→ $\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = 1 = D$

→ $D_x = \begin{vmatrix} 7 & 1 \\ 18 & 3 \end{vmatrix} = 21 - 18 = 3$

→ $D_y = \begin{vmatrix} 1 & 7 \\ 2 & 18 \end{vmatrix} = 18 - 14 = 4$

→ $x = \dfrac{D_x}{D} = 3$

→ $y = \dfrac{D_y}{D} = 4$

ii) $x_1 - x_2 + x_3 = -5$
$2x_2 - x_3 = 2$
$2x_1 + 3x_2 = -3$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & -5 \\ 0 & 2 & -1 & 2 \\ 2 & 3 & 0 & -3 \end{array}\right]$$

→ a) Matrix inverse method

$$\rightarrow \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -1 \\ 2 & 3 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -5 \\ 2 \\ -3 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -1 \\ 2 & 3 & 0 \end{bmatrix}^{-1} \begin{bmatrix} -5 \\ 2 \\ -3 \end{bmatrix}$$

$$\Rightarrow \text{let } A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -1 \\ 2 & 3 & 0 \end{bmatrix}; \; |A| = 1(3) + 1(2) + 1(-4) = 3 + 2 - 4 = 1$$

$$\rightarrow M_{ij} = \begin{bmatrix} 3 & 2 & -4 \\ -3 & -2 & 5 \\ -1 & -1 & 2 \end{bmatrix}, \; C_{ij} = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}, \; adj(A) = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$$

$$\rightarrow A^{-1} = \frac{adj(A)}{|A|} = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} \begin{bmatrix} -5 \\ 2 \\ -3 \end{bmatrix} = \begin{bmatrix} -15+6+3 \\ 10-4-3 \\ 20-10-6 \end{bmatrix} = \begin{bmatrix} -6 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

b) Cramer's Rule

→ $D = 1$

$$\rightarrow D_{x_1} = \begin{vmatrix} -5 & -1 & 1 \\ 2 & 2 & -1 \\ -3 & 3 & 0 \end{vmatrix} = -5(3) + 1(-3) + 1(12) = -15 - 3 + 12 = -6$$

$$\rightarrow D_{x_2} = \begin{vmatrix} 1 & -5 & 1 \\ 0 & 2 & -1 \\ 2 & -3 & 0 \end{vmatrix} = 1(-3) + 5(2) + 1(-4) = -3 + 10 - 4 = 3$$

$$\rightarrow D_{x_3} = \begin{vmatrix} 1 & -1 & -5 \\ 0 & 2 & 2 \\ 2 & 3 & -3 \end{vmatrix} = 1(-12) + 1(-4) - 5(-4) = -12 - 4 + 20 = 4$$

→ $x_1 = \frac{D_{x_1}}{D} = -6$

→ $x_2 = \frac{D_{x_2}}{D} = 3$

→ $x_3 = \frac{D_{x_3}}{D} = 4$

(11) i) $\begin{vmatrix} x & y & z \\ 1 & 2 & 3 \\ -1 & 2 & 1 \end{vmatrix} = 0 \Rightarrow x(-4) - y(4) + z(4) = 0$

$-4x - 4y + 4z = 0$

$-x - y + z = 0$

iii) $\begin{vmatrix} x & y & z & 1 \\ 1 & 2 & 3 & 1 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 4 & 1 \end{vmatrix} = 0 \Rightarrow x\begin{vmatrix} 2 & 3 & 1 \\ 0 & 0 & 1 \\ 0 & 4 & 1 \end{vmatrix} - y\begin{vmatrix} 1 & 3 & 1 \\ 0 & 0 & 1 \\ -1 & 4 & 1 \end{vmatrix} + z\begin{vmatrix} 1 & 2 & 1 \\ 0 & 0 & 1 \\ -1 & 0 & 1 \end{vmatrix} -$

$-1\begin{vmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ -1 & 0 & 4 \end{vmatrix} = 0$

$\Rightarrow x\left[(-1(8)\right] - y\left[(-1(7)\right] + z\left[(-1(2)\right] - 0 = 0$

$-8x + 7y - 2z = 0$

(18) $x\,C_7H_8 + y\,HNO_3 \longrightarrow z\,C_7H_5O_6N_3 + w\,H_2O$

$\rightarrow$ $C \rightarrow 7x = 7z$ $\quad\rightarrow\quad 7x - 7z = 0$

$H \rightarrow 8x + y = 5z + 2w$ $\quad 8x + y - 5z - 2w = 0$

$N \rightarrow y = 3z$ $\quad y - 3z = 0$

$O \rightarrow 3y = 6z + w$ $\quad 3y - 6z - w = 0$

$$\left[\begin{array}{cccc|c} 7 & 0 & -7 & 0 & 0 \\ 8 & 1 & -5 & -2 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 3 & -6 & -1 & 0 \end{array}\right] \xrightarrow[R_1 \to \frac{1}{7}R_1]{R_2 \to R_2 - R_1} \left[\begin{array}{cccc|c} 1 & 0 & -1 & 0 & 0 \\ 1 & 1 & -2 & -2 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 3 & -6 & -1 & 0 \end{array}\right]$$

$$R_2 \to R_2 - R_1 \left[\begin{array}{cccc|c} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 3 & -2 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 3 & -6 & -1 & 0 \end{array}\right] \quad R_3 \to R_3 - R_2 \left[\begin{array}{cccc|c} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 3 & -2 & 0 \\ 0 & 0 & -6 & 2 & 0 \\ 0 & 3 & -6 & -1 & 0 \end{array}\right]$$

$$\xrightarrow[R_4 \to \frac{1}{3}R_4]{R_3 \to -\frac{1}{6}R_3} \left[\begin{array}{cccc|c} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 3 & -2 & 0 \\ 0 & 0 & 1 & -\frac{1}{3} & 0 \\ 0 & 1 & -2 & -\frac{1}{3} & 0 \end{array}\right] \quad R_4 \to R_4 - R_2 \left[\begin{array}{cccc|c} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 3 & -2 & 0 \\ 0 & 0 & 1 & -\frac{1}{3} & 0 \\ 0 & 0 & -5 & \frac{5}{3} & 0 \end{array}\right]$$

$-\frac{1}{3} + 2 \quad \frac{6-1}{3}$

$\Rightarrow z - \frac{1}{3}w = 0$ $\quad\quad\rightarrow x - z = 0$

$z = \frac{1}{3}w$ $\quad\quad x = z = \frac{1}{3}w$

$\rightarrow y + 3z - 2w = 0$

$y = -3z + 2w$

$= -3(\frac{1}{3}w) + 2w = -w + 2w = w$

7.428 + 2.571

4.285

5

$\Rightarrow$ Let $w = 3$ $\quad y = w = 3$

$z = 1$ $\quad x = z = 1$

$\Rightarrow$ The blanced eqn will be:

$C_7H_8 + 3HNO_3 \longrightarrow C_7H_5O_6N_3 + 3H_2O$

(19) i) $4h + 1.5c = 5(2)$ $\quad\quad$ ii) $2.5c = 2(2.5) + 5h$

$4h + 1.5c = 10$ $\quad\quad 2.5c = 5 + 5h$

$c = 2 + 2h$

$2h - c = -2$

$$\rightarrow \left[\begin{array}{cc|c} 4 & 1.5 & 10 \\ 2 & -1 & -2 \end{array}\right]$$

$$\xrightarrow[R_2 \rightarrow \frac{1}{2}R_2]{R_1 \rightarrow \frac{1}{4}R_1} \left[\begin{array}{cc|c} 1 & 0.375 & 2.5 \\ 1 & -0.5 & -1 \end{array}\right] \quad R_2 \rightarrow R_2 - R_1 \left[\begin{array}{cc|c} 1 & 0.375 & 2.5 \\ 0 & -0.875 & -3.5 \end{array}\right]$$

$$R_2 \rightarrow \frac{-1}{0.875} R_2 \left[\begin{array}{cc|c} 1 & 0.375 & 2.5 \\ 0 & 1 & 4 \end{array}\right] \rightarrow \begin{array}{l} c = 1.714\,kg \\ h + (0.375)(1.714) = 2.5 \\ h = 2.5 - 0.643 \\ h = 1.857\,kg \end{array}$$

$\Rightarrow c = 4kg$

$\rightarrow h + (0.375)(4) = 2.5$

$h = 2.5 - 1.5$ $\quad\quad \Rightarrow c = 4kg$ & $h = 1kg$

$h = 1kg$

(20) $I_1 + I_3 = I_2$ $\Rightarrow$ $I_1 - I_2 + I_3 = 0$ $\rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 3 & 2 & 0 & 7 \\ 0 & 2 & 4 & 8 \end{array}\right]$

$3I_1 + 2I_2 = 7$ $\quad\quad 3I_1 + 2I_2 = 7$

$2I_2 + 4I_3 = 8$ $\quad\quad 2I_2 + 4I_3 = 8$

$$\xrightarrow[R_3 \to \frac{1}{2}R_3]{R_2 \to R_2 - 3R_1} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 5 & -3 & 7 \\ 0 & 1 & 2 & 4 \end{array}\right] \xrightarrow{R_2 \to \frac{1}{5}R_2} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & -3/5 & 7/5 \\ 0 & 1 & 2 & 4 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 - R_2} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & -3/5 & 7/5 \\ 0 & 0 & \frac{13}{5} & 13/5 \end{array}\right] \Rightarrow \frac{13}{5} I_3 = \frac{13}{5} \Rightarrow \underline{\underline{I_3 = 1A}}$$

$\Rightarrow I_2 - \frac{3}{5} I_3 = \frac{7}{5}$

$I_2 = \frac{7}{5} + \frac{3}{5} = \underline{\underline{2A}}$

$\Rightarrow I_1 - I_2 + I_3 = 0$

$I_1 = I_2 - I_3$

$I_1 = 2 - 1 = \underline{\underline{1A}}$

(X) Question no 18 is not done since the electrical circuit diagram is not clear to see. But if so, it can be easily be done by forming linear equations using Kirchhoff's junction (point) rule and solving those equations as it is done in question no 20.